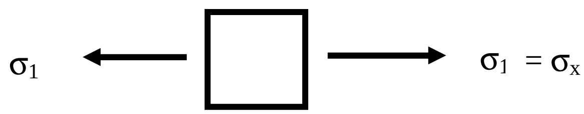
Strength Theories

The majority of material strength data is based on uniaxial tensile test results. Usually, all that you have to work with is the yield strength S_y and/or the ultimate tensile strength S_u.

This is fine if you only have the one normal stress component present : this is true for simple tension or compression members and for parts loaded only in bending.

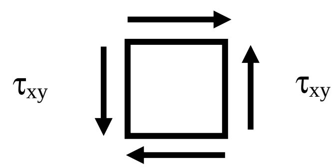


In this case, failure (defined as the onset of plastic deformation) occurs when
$\sigma_x = \sigma_1 = S_y/n$

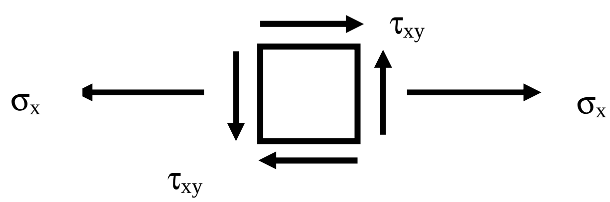
'n' is the factor of safety.

In many loading cases, we have more than just one normal stress component.

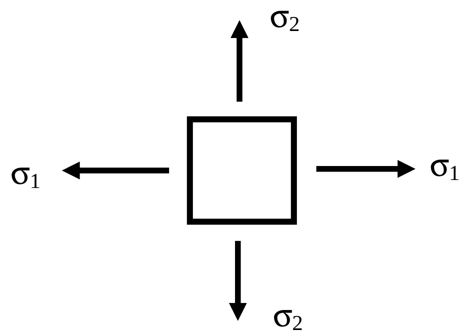
E.g. in torsion, we have a single shear stress component:



Or, combined bending and torsion in a shaft:



These cases can all be reduced to a simple biaxial case by finding the principal stresses, σ_1 and σ_2



Now when does failure occur? For ductile materials there are two commonly used strength theories - the Maximum Shear Stress (MSS) or Tresca theory and the von Mises or Distortion Energy theory.

Strength Theories

1. Maximum Shear Stress:

   This states that failure occurs when the maximum shear stress in the component being designed equals the maximum shear stress in a uniaxial tensile test at the yield stress:

   This gives $\tau_{max} = S_y/2n$ or

   $$|\sigma_1 - \sigma_2| = S_y/n$$
   or $$|\sigma_2 - \sigma_3| = S_y/n$$
   or $$|\sigma_3 - \sigma_1| = S_y/n$$

   whichever of the last three leads to the safest result. The latter usually involves σ_3 being zero, i.e. plane stress, and both σ_1 and σ_2 having the same sign. Note that the yield strength is reduced by the factor of safety 'n'.

2. von Mises or Distortion Energy Theory:

   This states that failure occurs when the von Mises stress σ_e in the component being designed equals the von Mises stress σ_e in a uniaxial tensile test at the yield stress:

This gives: $\sigma_e = \sqrt{2}/2\,[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{0.5} = S_y/n$

   In the plane stress case we have $\sigma_3 = 0$ and hence:

   $$\boldsymbol{\sigma_e = [\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2]^{0.5} = S_y/n}$$

   This is the most commonly used of the strength equations.

A third theory, the Maximum Normal Stress theory is similarly defined. It must NEVER be used for design with ductile materials. A modified version of this theory is sometimes used with brittle materials.

All three of these theories are shown on a plot the σ_1 versus σ_2 below:

Strength Theories